10. Non-steady Flame Propagation. Oxford: Pergamon Press, 1966.
11. MICHELSON D.M., SIVASHINSKY G.I., Thermal Expansion Induced Cellular Flames. Combust. and Flame, Vol.48, No.2, 1982.
12. BATEMAN H. and ERDELYI A., Higher Transcendental Functions. Vol.2, McGraw-Hill, New York, 1953.
13. POLYAKOV A.M., Isomeric States of Quantum Fields. Zh. eksperim. i teor. fiz., Vol.68, No.6, 1975.

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# THE THERMAL PROBLEM FOR A SUBMERGED STREAM*

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The general solution of the thermal problem of convective heat conduction with volume heat dissipation caused by viscous dissipation of kinetic energy of the fluid, whose velocity field is determined by the exact solution /1/ of the Navier-Stokes equations, is considered for a submerged stream. The possible formulation of the heat problem and the characteristic behaviour of the solutions are investigated. The solutions obtained have a special feature, namely the existence, under specified conditions, of two regimes of convective heat exchange.

A particular solution of the problem in question corresponding to a point heat source superimposed on the steam source was obtained in /2/, without taking into account the dissipative heat emission. The solution corresponds to the first term of the expansion of the temperature in a series in multipoles

$$T(R,\ \theta)=\sum_{n=1}^{\infty}\tau_n(\theta)\,R^{-\alpha_n} \tag{1}$$

where $R, \theta$ are spherical coordinates and the angle $\theta$ is measured from the stream axis. The appearance in (1) of the fractional indices $\alpha_n$ is connected with the presence, in the equation of heat conduction

$$LT=\frac{\nu}{2c_p}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)^2,\quad L=(u,\ \nabla)-a\Delta \tag{2}$$

of the convective term which changes the spectrum of the operator $L$.

The velocity field for a submerged stream has the form /1/

$$u_R=\nu\frac{y'(x)}{R},\quad u_\theta=\nu\frac{y(x)}{\sqrt{1-x^2}}\frac{1}{R};\quad y(x)=-2\frac{1-x^2}{A-x},\qquad x=\cos\theta \tag{3}$$

The coefficients of kinematic viscosity $\nu$ and thermal conductivity $a$ are assumed to be constant, $A>1$ is a constant connected monotonically with the momentum of the stream

$$I=16\pi\rho\nu^2A\left[1+\frac{4}{3(A^2-1)}-\frac{A}{2}\ln\frac{A+1}{A-1}\right] \tag{4}$$

and according to (4) $I\to 0$ as $A\to\infty$ and $I\to\infty$ as $A\to 1$.

The expansion (1) holds only for the solution of the homogeneous equation (2). The solution of the inhomogeneous equation of heat conduction (2) contains, apart from (1), a term whose form is determined by the dissipative heat source. Without the convective terms in the homogeneous equation the expansion (1) assumes the classical form, with $\alpha_n=n$ and $\tau_n$ being spherical functions. In the general case $\alpha_n\neq n,\ n>1$. According to (3), the dissipative function in (2) is proportional to $R^{-4}$, and this generates in (1), with one exception which will be noted below, an additional term of the form $z(x)R^{-2}$ corresponding to the particular solution of the inhomogeneous equation.

Substituting (1) and (3) into (2), we obtain for $\alpha_n\neq 2$

$$(1-x^2)\tau_n''-2x\tau_n'+\Pr(y\tau_n'+\alpha_ny'\tau_n)+\alpha_n(\alpha_n-1)\tau_n=0 \tag{5}$$

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The particular solution of the inhomogeneous equation corresponds to the value $\alpha_n = 2$ and satisfies the equation

$$(1 - x^2) u'' - 2xu' + \Pr (yu' + 2y'u) + 2u + \Phi (x) = 0 \tag{6}$$

$$\Phi(x) = -\frac{(A^2-1)^3}{(A-x)^6} + \frac{2A(A^2-1)^2}{(A-x)^5} + \frac{2(A^2-1)^2}{(A-x)^4} - \frac{5A(A^2-1)}{(A-x)^3} + \frac{A^2}{(A-x)^2} + \frac{A}{A-x}; \quad z = \frac{16 \Pr \nu^2}{c_p} u, \quad \Pr = \frac{\nu}{a} \tag{7}$$

Here $\Phi(x)$ is a normalized dissipative function and $c_p$ is the heat capacity.

Since the stream axis $(x = \pm 1)$ belongs, when $R \neq 0$, to the region of flow, it follows that we should demand that the solutions of (5) and (6) be bounded at the points $x = \pm 1$ at which the velocity field (3) is analytic. The linear equations (5), (6) belong to the Fuchs class /3/, and since the defining equations have double zero roots at the points $x = \pm 1$, it follows that in the neighbourhood of each of the singularities $x = \pm 1$ one of the solutions of each equation is analytic, and the other has a logarithmic singularity.

Thus the solutions behave like Legendre functions. For the analytic solutions we have $(1 - x^2)\tau_n'' \to 0$ as $x \to \pm 1$. This yields the conditions (only the upper or lower plus and minus signs are taken)

$$2\tau_n' (\pm 1) \mp [\Pr \alpha_n y' (\pm 1) + \alpha_n (\alpha_n - 1)] \tau_n (\pm 1) = 0 \tag{8}$$

$$u' (\pm 1) \mp [\Pr y' (\pm 1) + 1] u (\pm 1) \mp {}^1/_2 \Phi (\pm 1) = 0 \tag{9}$$

We note that the condition that the derivatives $\tau_n' (\pm 1)$, $u' (\pm 1)$ be bounded leads to the following relations:

$$d\tau_n/d\theta = du/d\theta \,|_{\theta = 0,\, \pi}$$

corresponding to the axial symmetry of the temperature field. Problem (5), (8) is a problem in the eigenvalues $\lambda_n = \alpha_n (\alpha_n - 1)$ and the corresponding eigenfunctions $\tau_n$ of the multipole expansion (1). Here $\tau_n$ are found apart from arbitrary constant multipliers $C_n$, whose assignment determines the structure and intensity of the thermal singularity at the point $R = 0$. We see that when $\alpha_1 = 1$, the eigenvalue of the problem is $\lambda_1 = 0$ and the corresponding eigenfunction is

$$\tau_1 (x) = C_1 \varphi (x), \quad \varphi (x) = (A - x)^{-2\Pr} \tag{10}$$

The solution (10) obtained in /2/ corresponds to a heat source whose given intensity determines the constant $C_1$.

Instead of specifying the intensities $C_n$, we could e.g. specify for problems (5), (8) an arbitrary temperature field $T(R_0, \theta)$ on a sphere of radius $R_0$. Then the coefficients $C_n$ would be uniquely determined in the case of a complete system of linearly independent functions.

Equation (5) is the result of the action of a non-selfconjugate generalized differential Legendre operator /4/ on $\tau_n$, whose spectrum consists of isolated eigenvalues without the limit points. The linear envelope of the eigenfunctions of this operator is dense in $L_2((-1, 1))$ /4/. Consequently the system of functions $\tau_n$ is complete in $L_2([-1, 1])$, since it allows the analytic continuation of the points $x = \pm 1$.

The eigenvalues $\lambda_n = \alpha_n (\alpha_n - 1)$ of the differential operator in (5) are real, and the eigenfunctions are orthogonal, with weight $1/\varphi$.

Indeed, substituting $\tau_n = \varphi g_n$ into (5) we obtain $[(1 - x^2) \varphi g_n']' + (\alpha_n - 1) \Pr y' \varphi g_n + \lambda_n \varphi g_n = 0$, from which we obtain, using standard procedures,

$$[(1 - x^2) \varphi (g_m g_n' - g_n g_m')] \,|_{-1}^{1} + \Lambda = 0$$

$$\Lambda = (\lambda_n - \lambda_m) \int_{-1}^{1} \varphi g_n g_m \, dx$$

Since $\varphi, g_n, g_m$ are analytic at the points $x = \pm 1$, therefore $\Lambda = 0$ and this implies the orthogonality of the system of functions $\tau_n$ when $n \neq m$, as $\lambda_n \neq \lambda_m$.

Replacing $\tau_m$ by $\tau_n^*$ and $\lambda_m$ by $\lambda_n^*$ respectively (the asterisk denotes the complex conjugate), we obtain from the condition $\Lambda = 0$, $(\tau_n \not\equiv 0)$ $\lambda_n = \lambda_n^*$, i.e. all eigenvalues of the non-selfconjugate differential operator of the equation (5) are real.

The eigenvalues $\lambda_n$ and $\alpha_n$ depend on two parameters, the momentum of the stream and the Prandtl number. Clearly, when $\Pr = 0$ or $I \to 0$, the convective terms in (5) vanish and $\alpha_n = n$, while the corresponding $\lambda_n = n(n - 1) \geqslant 0$ for any integer value of $n$.

We note that when $\Pr$ are arbitrary and $A$ $(>1)$, all eigenvalues $\lambda_n (\Pr, A) \geqslant 0$ since the smallest eigenvalue at all $\Pr$ and $A$ is $\lambda_1 = 0$. This follows from the solution /2/ and the fact that all eigenvalues continuous in $\Pr$ and $A$ are isolated. This in turn implies that the indices $\alpha_n$ in (1) are also isolated and real. From the physical point of view it means that the solutions of the homogeneous equation (2) with velocity field (3) are not of the wave type. It is convenient to replace the number $A$ or moment $I$ by the Reynolds number

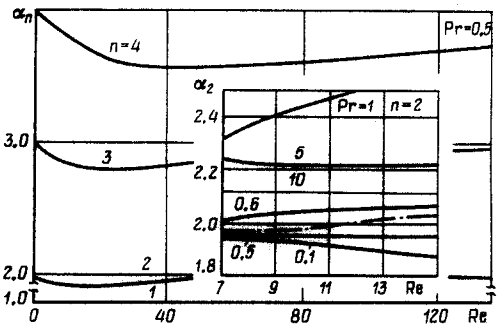


Fig.1

$$\mathrm{Re} = \left(\frac{I}{\pi\nu^2}\right)^{1/2}$$

The relations $\alpha_n = \alpha_n(\mathrm{Pr}, \mathrm{Re})$ obtained by numerical methods (described below) are shown in Fig.1, for $n = 1, 2, 3, 4$ and $\mathrm{Pr} = 0.5$, and also for $n = 2$ and various values of Pr. We note that the curves $\alpha_2(0.5; \mathrm{Re})$ and $\alpha_2(0.6; \mathrm{Re})$ lie on different sides of the horizontal line $\alpha_2 = 2$. When $0.5 < \mathrm{Pr} < 0.6$, the curves showing the dependence of $\alpha_2$ on Re intersect this horizontal, and this is shown for the case $\mathrm{Pr} = 0.54$ with a dash-dot line. This will be important in what follows.

Thus, problem (5),(8) has a solution for all admissible values of the numbers Pr and Re. The solution of the homogeneous Eq.(2) therefore exists and is unique in the form (1), at least for a heat problem outside a sphere of radius $R_0$, with any type of thermal boundary conditions at its surface and zero (or finite and constant) temperature at infinity.

In the case of a thermal problem in a spherical layer, series (1) must be supplemented by terms with positive powers of $R$. This is met by writing the solution in the form

$$T(R, \theta) = \sum_{n=-\infty}^{\infty} \tau_n(\theta) R^{-\alpha_n}$$

A solution of this problem clearly also exists and is unique, provided that we specify any thermal conditions at the boundaries of the spherical layer. Evidently, the existence and uniqueness of the solution of the boundary value problem for the homogeneous equation of convective heat conduction will occur in the case of the Laplace equation, and for regions of more general type.

We have a different situation in the case of the inhomogeneous problem (6),(9), in which the dissipative heat emission is taken into account. If we write formally in Eq.(5) $\alpha_n = 2$, it will be identical with Eq.(6) in which $\Phi(x) \equiv 0$.

Computations show that at definite values of Pr and Re problem (5),(8) has $\lambda_n = 2$ as the eigenvalue corresponding to $\alpha_n = 2$. In these cases the solvability of the inhomogeneous problem beomes questionable. We can show analytically, using the Fredholm alternative, that problem (6),(9) has no solution as $\mathrm{Re} \to \infty$, provided that $\mathrm{Pr} = 1/2$.

When the values of the Re number are arbitrary, we have a curve $\mathrm{Pr}_* = \mathrm{Pr}(\mathrm{Re})$ on which the boundary value problem (6),(9) has no solution. The curve will be computed by numerical methods.

The computations were carried out as follows. We posed two Cauchy problems for the homogeneous Eq.(6)

$$u_+(1),\ u_-(-1),\ u_\pm'(\pm 1) \qquad (9) \qquad \Phi(\pm 1) = 0$$

The solution was obtained using the Runge-Kutta-Mercen method with a relative error per step of $10^{-6}$. The integration was carried out up to some point of matching $x_c$ lying in the region of steepest gradients. The solution $u_-(x)$ is analytic near the point $x = -1$ and becomes, generally speaking, unbounded when $x = 1$. If however a set of parameters Pr and Re exists for which the solution $u_-(x)$ will also by analytic when $x = 1$, then the functions $u_-(x)$ and $u_+(x)$ will be identical, apart from constant multipliers, everywhere in the interval $[-1, 1]$. To make it happen for the solutions of the homogeneous second-order Eq.(6) with homogeneous boundary conditions (9), it is sufficient that the functions and their first derivatives agree at any point $x_c$ of the interval

$$Au_-(x_c) = Bu_+(x_c), \quad Au_-'(x_c) = Bu_+'(x_c)$$

The condition that the above system has a non-trival solution is, that the determinant

$$\Delta = \begin{vmatrix} u_-(x_c) & u_+(x_c) \\ u_-'(x_c) & u_+'(x_c) \end{vmatrix}$$

does vanish. If at some value of $\mathrm{Pr}(\mathrm{Re})$ the quantity $\Delta = 0$, then the problem of a solvability of the inhomogeneous problem is dealt with as follows. The solution of inhomogeneous Cauchy problems, where $\Phi(\pm 1)$ is obtained from (7), can be written for (6) in the form $Au_- + Bu_-^\circ$ and $Bu_+ + u_+^\circ$ where the superscript $^\circ$ denotes the particular solutions of the inhomogeneous equations. The necessary condition for these solutions to agree, and thus for the inhomogeneous problem to have a solution, is the vanishing of the determinant

$$\Delta_1 = \begin{vmatrix} u_-(x_c) & u_+^{\circ}(x_c) - u_-^{\circ}(x_c) \\ u_-'(x_c) & u_+^{\circ\prime}(x_c) - u_-^{\circ\prime}(x_c) \end{vmatrix} \tag{11}$$

provided that $\Delta = 0$. If $\Delta_1 \neq 0$, then the inhomogeneous problem has no solution.

Figure 2 shows the results of the computations in the form of the relation $\mathrm{Pr}_* = \mathrm{Pr}\,(\mathrm{Re})$ in case when $\Delta = 0$. We see that the numerical computations correspond to the asymptotic analysis for the case $\mathrm{Re} \to \infty$. We note that the same algorithm was used to determine the eigenvalues $\alpha_n\,(\mathrm{Pr},\,\mathrm{Re})$ of (5), representing the roots of the equation $\Delta\,(\alpha_n) = 0$.

It should be stressed that the lack of a solution to problem (6),(9) does not imply the insolubility of the initial stationary problem of convective heat conduction with viscous heating (2), provided that the velocity field is given in the form (3). On the insolubility curve $\mathrm{Pr}_* = \mathrm{Pr}\,(\mathrm{Re})$ the index of the dipole term in (1) $\alpha_2\,(\mathrm{Pr},\,\mathrm{Re})$ and the solution of the inhomogeneous equation proportional to $R^{-2}$ interact with the solution of the homogeneous equation in such a manner, that the dependence of $R$ on the inhomogeneous equation becomes different; thus problem (6),(9) loses its meaning on the curve $\mathrm{Pr}\,(\mathrm{Re})$ where $\alpha_2\,(\mathrm{Pr},\,\mathrm{Re}) = 2$.

In this case we shall seek the particular solution of the inhomogeneous Eq.(2) in the form

$$T\,(R,\ x) = \frac{u_0(x)}{R^2} + C\,\frac{u_1(x)}{R^2}\ln R, \quad C = \mathrm{const} \tag{12}$$

which is dictated by the power dependence of the solution of the homogeneous equation and heat source in (2), on $R$. Substituting (12) into (2) and taking (3) into account, we obtain

$$[(1 - x^2)\,u_0']' + 2u_0 + \mathrm{Pr}\,(2y'u_0 + yu_0') = (3 + \mathrm{Pr}\,y')\,Cu_1 - \Phi\,(x) \tag{13}$$

$$[(1 - x^2)\,u_1']' + 2u_1 + \mathrm{Pr}\,(2y'u_1 + yu_1') = 0 \tag{14}$$

The boundary conditions for (13) follow from (9) after the substitutions $u \to u_0$, $\Phi \to \Phi - (3 + \mathrm{Pr}\,y')\,Cu_1$, and for (14) from (8) after the substitution $\tau_n \to u_1$, $\alpha_n \to 2$ and the assumption that $u_1\,(1) = 1$. The constant $C$ is chosen from the condition for (13) $\Delta_1\,(x_c) = 0$ (11) to be solvable.

The constant $C$ does not depend on the matching point $x_c$, since we can find $C$ in the same manner from the condition that the solution of the equation conjugate to (14) is orthogonal to the right-hand side of (13). The fact that the solution of the conjugate equation is not trivial follows from the insolubility of problem (6),(9) and the Fredholm alternative. The solutions (13) and (14) are shown in Fig.3 for certain values of $\mathrm{Pr}\,(\mathrm{Re})$, and we have put there $u_1\,(1) = 1$. Curves *1* correspond to $\mathrm{Pr} = 0.56$, $\mathrm{Re} = 7.08$; *2* — $\mathrm{Pr} = 0.55$, $\mathrm{Re} = 9.23$; *3* — $\mathrm{Pr} = 0.54$, $\mathrm{Re} = 12.1$; *4* — $\mathrm{Pr} = 0.53$, $\mathrm{Re} = 16.3$.

A special feature of the solutions of (7) is the presence of a region in which $u\,(x) < 0$. Thus the viscous heating leads to the appearance of a negative contribution towards the temperature, stipulated by the dipole character of the term with $n = 2$ in expansion (1).

Indeed, in the limit when $\mathrm{Pr} = 0$, we have $u\,(x) = Cx$ so that $u\,(x)$ change their sign when $x = 0$.

Let us consider the case of $\mathrm{Pr} \to \infty$. We shall consider the solution of (7) near the point $x = 1$. Putting $t = 1 - x \ll 1$, $\mu = 2\mathrm{Pr}/(A - 1)$ we obtain from (6)

$$tu'' + (1 + \mu t)\,u' + 2\mu u + \Phi\,(1)/2 = O\,(1/\mu,\ t),\ u = u\,(t)$$

The solution of this equation has the following form, up to second-order infinitesimals:

$$u\,(t) = C\,(1 - \mu t)\,e^{-\mu t} - \Phi\,(1)/(4\mu),\ C = \mathrm{const}$$

From this we have $u < 0$ when $t = 1/\mu$, therefore by virtue of the continuity there exists a domain of values of $x$ where $u < 0$.

In the case of arbitrary $\mathrm{Pr}$ and $\mathrm{Re}$ we obtained the solution of (6) by numerical methods. Figure 4 shows characteristic profiles of $u\,(x)$ near the curve of insolubility. The number $\mathrm{Pr}$ serves as a parameter with $\mathrm{Re} = 15.2$ fixed. Curves *1*–*6* correspond to the values of the Prandtl number equal to $0.525$; $0.529$; $0.531$; $0.533$; $0.535$; $0.539$. We see that when the solution passes through the value $\mathrm{Pr}_*$ lying on the insolubility curve, it undergoes a qualitative change and the physical meaning of the solutions also becomes different.

We note that a solution of type (12) is complete for the problem in question, since the eigenvalues of the homogeneous problem are isolated. If any of the eigenvalues were repeated, then we would have to add in (1), as well as in (12), terms in the form of the $k - 1$-th degree polynomials in $\ln R$ where $k$ is the multiplicity, multiplied by $R^{-\alpha_n}$ where $\alpha_n$ determines the $k$-tuple eigenvalue $\lambda_n = \alpha_n\,(\alpha_n - 1)$.

To explain the physical meaning of the insolubility curve, we shall consider a simple example of a thermal problem of convective heat conduction with viscous heating for a plane hydrodynamic sink.

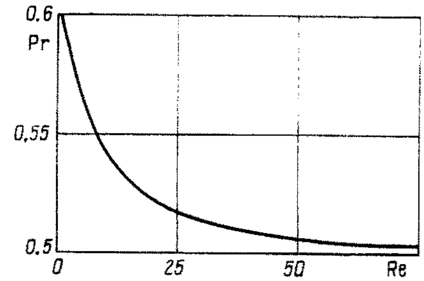


Fig.2

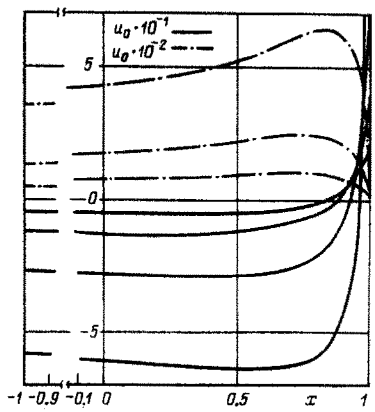


Fig.3

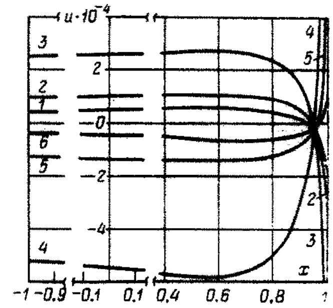


Fig.4

The heat conduction Eq.(2) of the axisymmetric problem with velocity field

$$u_r = -\frac{Q}{2\pi r}, \quad u_\theta \equiv 0$$

where $(r, \theta)$ are polar coordinates and $Q$ denotes the consumption ( Pe is the Peclet number), takes the form

$$-\frac{\mathrm{Pe}}{r}\frac{dT}{dr} = \frac{1}{r}\frac{d}{dr}r\frac{dT}{dr} + \frac{q}{r^4}, \quad \mathrm{Pe} = \frac{Q}{2\pi a}, \quad q = \frac{4\nu a\,\mathrm{Pe}^2}{c_p} \tag{15}$$

The solution of (15) with boundary conditions $T(r_0) = T_0$, $T(\infty) = 0$ has the form

$$T(r) = \left[T_0 - \frac{q}{2(\mathrm{Pe}-2)}\frac{1}{r_0^2}\right]\left(\frac{r}{r_0}\right)^{-\mathrm{Pe}} + \frac{q}{2(\mathrm{Pe}-2)}\frac{1}{r^2}, \quad \mathrm{Pe} \neq 2 \tag{16}$$
$$T(r) = q\frac{\ln(r/r_0)}{2r^2} + T_0\frac{r_0^2}{r^2}, \quad \mathrm{Pe} = 2$$

We see that the solution has the same character as that in the case of the Landau jet. The point $\mathrm{Pe} = 2$ corresponds to the insolubility curve. When $\mathrm{Pe} \to \infty$, solution (16) implies that the term $\sim r^{-\mathrm{Pe}}$ characterizes the thermal boundary layer at $r = r_0$. When $\mathrm{Pe} = \infty$, the thermal problem becomes a problem of convective heat transfer without thermal diffusion. The boundary conditions for this problem can only be posed at the boundary of the fluid inflow, and in this case the condition is $T(\infty) = 0$. At large, but finite values of Pe, convective heat transfer prevails over conductive transfer, so that the conditions at the outflow boundary $(r = r_0)$ are not of major importance. This region of convective heat transfer corresponds to $\mathrm{Pe} > 2$.

When $\mathrm{Pe} < 2$, the formulation of the boundary condition at $r = r_0$ is of major importance, since when $r \to \infty$, the term $\sim r^{-\mathrm{Pe}}$ becomes dominant and determines the temperature of the fluid at the inflow region. In this case the boundary conditions interact, and this represents a characteristic feature of the heat transfer by diffusion. The boundary $\mathrm{Pe} = 2$ separates these two characteristic modes of the heat transfer.

Note that the solution of the problem in a region with the point $r = 0$ removed, has a physical meaning only when $\mathrm{Pe} > 2$. When $\mathrm{Pe} < 2$ and $r \to 0$, the non-physical negative temperatures obtain.

In the case of a Landau jet the physically admissible solutions can be obtained only in the region $0 < R_0 < R$, since a region with $u(x) < 0$ always exists. Just as in the case of a plane hydrodynamic sink, the insolubility curve for the problem (6),(9) $\mathrm{Pr}_* = \mathrm{Pr}(\mathrm{Re})$

separates the region of the primary convective heat transfer $(\mathrm{Pr} > \mathrm{Pr}_*)$, from the region of the primary conductive heat transfer $(\mathrm{Pr} < \mathrm{Pr}_*)$.

Indeed, if the thermal flux at infinity is zero, then $\tau_1 \equiv 0$, and in the case of $\mathrm{Pr} > \mathrm{Pr}_*$ the principal term, as $R \to \infty$, represents the particular solution of the inhomogeneous equation (2): $z(x) R^{-2}$, and the temperature at infinity will be determined by the dissipative heating and not by the heat source on the sphere $R = R_0$. If on the other hand $\mathrm{Pr} < \mathrm{Pr}_*$, then the dipole term of the solution of the homogeneous Eq.(2) will become principal as $R \to \infty$, i.e. in this case the influence of the boundary condition at $R = R_0$ will extend to infinity, the latter effect being characteristic for conductive heat conduction.

If the heat flux at infinity is not zero, then it hardly makes sense to distinguish between those two modes of heat transfer. It should be noted that the possibility of separating the heat transfer modes depends essentially on the manner in which the heat source, which in the present case is the viscous dissipation of kinetic energy of the fluid, is distributed throughout the volume.

## REFERENCES

1. LANDAU L.D., On a new exact solution of the Navier-Stokes equations. Dokl. Akad. Nauk SSSR, Vol.43, No.7, 1944.
2. RUMER YU.V., Convective diffusion in a submerged stream. PMM Vol.17, No.6, 1953.
3. SMIRNOV V.I., Course of Higher Mathematics. Vol.3, Pt.2, Moscow-Leningrad, Gostekhizdat, 1953.
4. TRIBEL' KH., Theory of Interpolation, Functional Spaces, Differential Operators. Moscow, Mir, 1980.

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# ON THE NON-LINEAR MAXWELL-TYPE DEFINING EQUATIONS FOR DESCRIBING THE MOTIONS OF POLYMER LIQUIDS*

A.N. PROKUNIN

The problem of using non-linear Maxwell equations containing elastic deformation as the intermediate parameter, to approximate experimental data on the motion of polymer liquids under arbitrary elastic deformations, is studied. The data used concern the simple shear, uniaxial tension and pure shear. The form of the dependence of the free eneryg and rate of irreversible deformation on the elastic deformation is expressed in specific terms. It is assumed that in the course of the deformation, directional phenomena such as crystallization and mechanical destruction play a negligible part. Maxwellian models where the total deformation is separated into the elastic and the irreversible part, were constructed in /1-12/**, initially in the region of small elastic deformations /1-3/. (**See also: Kuvshinskii E.V. Study of the flows of macropolymer solutions (Mechanics of Elastic and Viscoelastic Media). Dis. na soiskanie uch. st. dokt.fiz.matem.nauk. Leningrad, Leningr.fiz.-tkhn.in-t,1950; Leonov A.I. On the description of rheological behaviour of viscoelastic media under large elastic deformations. Preprint In-ta problem mekhan. Akad. Nauk SSSR, Moscow, No.34, 1973; Leonov A.I. Non-equilibrium thermodynamics and rheology of viscoelastic polymer media. Preprint lektsii prochitannoi v Mezhdunarodnoi shkole "Problemy teplo- i massoperenosa v reologicheski slozhnykh sredakh "Minsk, 1975).

The present paper deals with the basic propositions developed in /7, 8,10/ for Maxwell media. Basically, we use the approach of /7/ in which the general form of Maxwell's equations is obtained within the framework of quasilinear non-equilibrium thermodynamics under the assumption that the locally equilibrium state of the medium is non-linearly elastic. The drawbacks apparent in the description of the experimental facts, based on specific equations, are noted and a simple method for overcoming them is proposed.

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